developed in between the hole locations at the natural spacing, and a natural wave family also persisted downstream of the holes.

Probable Causes of Surface Pattern Mechanism

A detailed listing of ablation pattern observations is given in tabular form by Laganelli and Nestler[1] with a statement of possible inference that each observation suggests regarding the pattern formation mechanism. The authors suggest a potential relationship between longitudinal vorticity and the diamond-shape pattern which has been explored in Refs. 5 and 7. On the other hand, the periodic behavior of the pattern phenomenon suggests a preferred modal or resonant condition resulting from the fluctuating components in a turbulent flow which was the subject of the investigations in Refs. 2–4, 6, and 8. The experimental results of Ref. 10 would tend to give support to this argument where the pattern wavelength was observed to change during the test. Further support was observed in the Langley tests where induced disturbances at half and twice the natural wavelength developed patterns which diverged and converged, respectively, to their natural mode. It should be noted that the analytical concepts require the existence of vorticity to initiate the modal response or pressure waves from disturbances. Furthermore, it has been observed that the transverse wavelength of longitudinal grooves (preceding diamond patterns) as developed from various vortex generating mechanisms has typically the same value as the diamond pattern. The spacing ratio λ/δ appears to be in good agreement with the range of λ/δ for longitudinal vortices; hence, the significant role of longitudinal vorticity in diamond-pattern development is suggested.

It has been mentioned that although ablation is not a necessary requisite for pattern development, it is required for the patterns to enhance their features in a responding surface. An ablation age parameter,[11] defined as the ratio of the amount of material ablated to the boundary-layer thickness ($\Delta y/\delta$) appears to approach unity for the diamond-shaped patterns to develop in a uniform, symmetric matrix. This phenomenon was observed on the test cones (Figs. 2 and 3), recovered flight vehicles, and several of the rocket exhaust models of Ref. 1 which exhibited an ablation age parameter 0.4.

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Shapes of Missiles of Minimum Ballistic Factor

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A KEEN interest has been shown by a number of authors on the problem of determination of slender axisymmetric power law bodies having minimum ballistic factor. Berman[1] studied the problem for given values of body length, diameter, and specific weight and compared the ballistic coefficient (reciprocal of ballistic factor) of the body with that of a sharp cone having the same dimensions. He employed wind-tunnel measurements and calculated the ratio of ballistic coefficient of the power law body to that of the sharp cone from an empirical variation of drag coefficient with power law exponent. He concluded that the maximum ballistic coefficient of the body can be increased by about 40% over that for a cone if the power law exponent has a value of 0.62. Fink[2] studied the same problem analytically employing the Newtonian theory as well as Newton-Busemann centrifugal theory and compared his results with those of Berman. But in Fink's theory no account was taken of the frictional effects. Miele and Huang[3] considered the effect of skin friction but only employed the Newtonian theory. In this Note the authors have used the more realistic Newton-Busemann centrifugal law and have compared their results with those of Miele and Huang. Earlier results[1–3] follow as particular cases of our analyses.

The ballistic factor of a missile of given length, thickness, and specific weight is proportional to the quality coefficient

$$I = I_1/I_2 \tag{1}$$

where I_1 and I_2 denote the integrals associated with drag and volume of the missile.

The general expression for the drag of a body in hypersonic flow at zero angle of attack when the surface-averaged skin-friction coefficient is constant is given by[4]

$$\frac{D}{4\pi q} = \int_0^l y\left(\dot{y}^3 + k\frac{y\dot{y}\ddot{y}}{2} + \frac{c_f}{2}\right)dx \tag{2}$$

where k is a factor equal to zero for Newtonian law and equal to unity for Newton-Busemann law.

The volume of the body is given by

$$V = \pi\int_0^l y^2 dx \tag{3}$$

We now investigate the power law bodies described by

$$y = (d/2)(x/l)^n \tag{4}$$

From Eqs. (2–4), we have

$$I_1 = \frac{4Dl^2}{\pi q d^4} = \frac{2n^3 + k(n^3 - n^2)}{4(2n-1)} + \frac{\alpha^3}{(n+1)} \tag{5}$$

$$I_2 = 4V/\pi d^2 l = 1/(2n+1) \tag{6}$$

where

$$\alpha = (4C_f)^{1/3}(l/d)$$

Therefore the 'quality coefficient' from Eq. (1) becomes

$$I = (2n+1)\left[\frac{2n^3 + k(n^3 - n^2)}{4(2n-1)} + \frac{\alpha^3}{(n+1)}\right] \tag{7}$$

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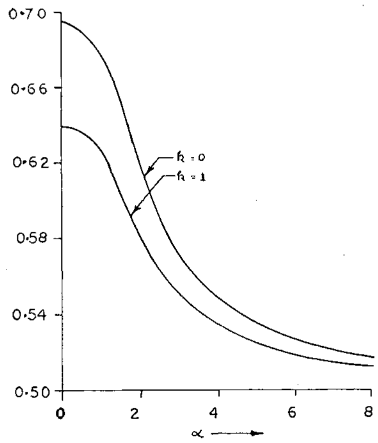

Fig. 1 Optimum power law exponent.

Now in order that the ballistic factor be minimum, $dI/dn = 0$, i.e.,

$$\frac{24n^4 - 8n^3 - 6n^2 + k(12n^4 - 12n^3 + n^2 + 2n)}{4(2n-1)^2} + \frac{\alpha^3}{(n+1)^2} = 0 \quad (8)$$

This gives the values of n for given values of the factor α. Knowing the value of n we can calculate the missile shapes of minimum ballistic factor and also from Eq. (7) the corresponding values of I.

Now Eq. (8) is a general expression and we can deduce all the previous results from here. The following four cases are possible. Case 1: When $k = 0$, $\alpha = 0$ (Newton law neglecting friction); $n = 0.694$ as also obtained by Fink.[2] Case 2: When $k = 1$, $\alpha = 0$ (Newton-Busemann law neglecting friction); $n = 0.637$ as also obtained by Fink.[2] Case 3: When $k = 0$, $\alpha \neq 0$ (Newton law accounting for friction); then the optimum value of n is given by

$$\frac{12n^4 - 4n^3 - 3n^2}{2(2n-1)^2} + \frac{\alpha^3}{(n+1)^2} = 0 \quad (9)$$

a result also obtained by Miele and Huang.[3] Case 4: When $k = 1$ and $\alpha \neq 0$ (Newton-Busemann law accounting for friction) is not reported so far and is being considered here. The results obtained are compared with those given by Case 3 due to Miele and Huang. In this case the relation between n and α is given by

$$\alpha = \left[\frac{(-36n^4 + 20n^3 + 5n^2 - 2n)(n+1)^2}{4(2n-1)^2}\right]^{1/3} \quad (10)$$

Fig. 1 compares the relationship $n(\alpha)$ for the cases when $k = 0$ and $k = 1$, respectively. Also Fig. 2 compares the corresponding relation $I(\alpha)$. Having found the value of n it is easy to find from Eq. (4) the geometry of the missiles of minimum ballistic factor for given values of α in both the cases $k = 0$ and $k = 1$. Table 1 gives the comparative values of n and I for $\alpha = 0,1,2$ for the two cases $k = 0$ and $k = 1$.

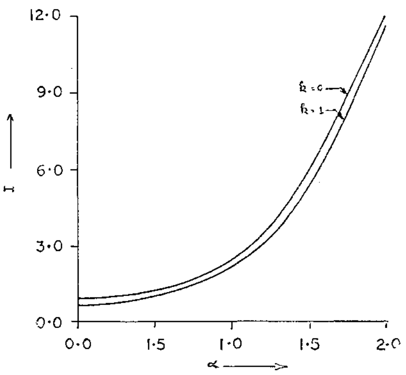

Fig. 2 Minimum quality coefficient

Table 1 Values of n and I for given values of α for the cases $k = 0$ and $k = 1$

	$k = 0$ (Newton Law) $\alpha = 0$	$\alpha = 1$	$\alpha = 2$	$k = 1$ (Newton-Busemann Law) $\alpha = 0$	$\alpha = 1$	$\alpha = 2$
n	0.6937	0.6780	0.6179	0.6372	0.6248	0.5822
I	1.0286	2.4354	12.1738	0.7670	2.1537	11.7767

It can be observed that with the increasing values of the friction factor α the power law exponent n decreases while the corresponding ballistic factor increases. Also the values of n and I obtained by using Newton-Busemann law are always less than those obtained by using Newton law corresponding to the same value of α.

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Flow through an Asymmetric Two-Dimensional Contraction

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A RECENT paper by Oates and Nash-Webber[1] considered the problem of an irrotational, incompressible flow in a two-dimensional channel with a contraction. The purpose of their study was to determine the nonuniformity of the axial velocity profile beyond the contraction. This is an important consideration when using such a contraction in the entrance region of a wind or water tunnel. The assumption made in Ref. 1 is that the amplitude of the contraction is small compared to the mean channel height. The problem considered in this paper is the same as above, but no restriction is imposed on the amplitude of the contraction.

The geometry of the channel is shown in Fig. 1, where $g(x)$ and $f(x)$ are continuous functions with continuous first derivatives, defined as

$$g(x) = \begin{cases} A, & x \le -L \\ \bar{g}(x), & -L \le x \le \alpha L \\ a, & x \ge \alpha L \end{cases}$$
$$f(x) = \begin{cases} -A, & x \le 0 \\ \bar{f}(x), & 0 \le x \le \alpha L \\ -b, & x \ge \alpha L \end{cases} \quad (1)$$

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